CyberChallenge.IT 2024 - Pretest
Commented solutions

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1 Question 1

1.1 Question

Four cards are on a table. On each card a character is written, respectively M, I, 9, 7. We also know that every card has a letter on one side, and a positive integer on the other one. Francesco says “If a card has an even number on one side, then it has a consonant on the other side”. Paolo disproves Francesco’s sentence by turning over a card. Which character is on the card turned over by Paolo?

1.2 Answers

(A) M

(B) I

(C) 9

(D) 7

1.3 Proposed solution

The correct answer is `(B) I`

To disprove Francesco, Paolo has to show a card that has an even number on one side and a vowel on the other side. Since M is a consonant and 9 and 7 are odd numbers, the only possible card is the one with the letter I on it.

2 Question 2

2.1 Question

Alice and Bob play a game. Alice chooses N distinct real numbers between 0 and 1000. Bob lists down the absolute difference of all the possible pairs (a, b) in Alice's set such that $a \neq b$.
Bob wishes to find 10 distinct numbers on his list not exceeding 100. What is the least value of N such that Bob can always accomplish this?

2.2 Answers

(A) 201

(B) 501

(C) 101

(D) There is no such N

2.3 Proposed solution

The correct answer is `(C) 101`.

Let $x_0, \ldots, x_{N-1}$ be the numbers chosen by Alice, in ascending order. Since these are all distinct, the following numbers in Bob's list are all different:

$$\begin{aligned} y_1 &= \min(x_1 - x_0, x_2 - x_1, \ldots, x_{N-1} - x_{N-2}); \\ y_2 &= \min(x_2 - x_0, x_3 - x_1, \ldots, x_{N-1} - x_{N-3}); \\ &\vdots \\ y_{10} &= \min(x_{10} - x_0, x_{11} - x_1, \ldots, x_{N-1} - x_{N-11}). \end{aligned}$$

Moreover it holds $y_1 < y_2 < \ldots < y_{10}$.
Now let's suppose that $N \geq 101$. In this case we have

$$1000 \geq x_{100} - x_0 = (x_{100} - x_{90}) + (x_{90} - x_{80}) + \ldots + (x_{10} - x_0) \geq 10 \cdot y_{10}.$$

Hence $y_{10} \leq 100$ and $y_1, \ldots, y_{10}$ are 10 distinct numbers on Bob's list not exceeding 100.
Now let's prove that, for $N = 100$, Alice is able to choose the numbers in such a way that Bob cannot find such 10 numbers. Indeed, Alice can choose the list

$$\left\{0, 10 + \frac{1}{100}, 20 + \frac{2}{100}, \ldots, 990 + \frac{99}{100}\right\}.$$

In this list, the difference between two consecutive numbers is always $10 + \frac{1}{100}$, thus

$$\begin{aligned} x_1 - x_0 = x_2 - x_1 = \ldots = x_{N-1} - x_{N-2} &= 10 + \frac{1}{100} \\ x_2 - x_0 = x_3 - x_1 = \ldots = x_{N-1} - x_{N-3} &= 20 + \frac{2}{100} \\ &\vdots \\ x_{10} - x_0 = x_{11} - x_1 = \ldots = x_{N-1} - x_{N-11} &= 100 + \frac{10}{100} > 100 \end{aligned}$$

and thus all the other differences are greater then 100. So Bob can find at most 9 distinct numbers on his list not exceeding 100.

3 Question 3

3.1 Question

A TeamItaly class has 42 people. All the 42 people are either tutors or students. The first tutor gives a challenge to 23 students, the second to 24 students, the third to 25 students and so on until the last tutor, which gives a challenge to all the students. How many students are there in the class?

3.2 Answers

(A) 26

(B) 28

(C) 32

(D) 36

3.3 Proposed solution

The correct answer is `(C) 32`.

Let x be the number of tutors and y the number of students, with $x + y = 42$. Since the first tutor assigns 23 challenges, the last one will assign $23 + x - 1$ challenges to the students, thus we know that $23 + x - 1 = y$. Creating a system we the first equation we obtain $x = 10$ and $y = 32$. Hence the answer is 32.

4 Question 4

4.1 Question

1000 students participated in the admission test for CyberChallenge.IT. Each of the 1000 participants scored an integer number of points between 0 and 15 (both included).
Given that the average score among all of them is 6, what is the maximum number of participants that could have scored exactly 15 points?

4.2 Answers

(A) 66

(B) 250

(C) 400

(D) 500

4.3 Proposed solution

The correct answer is `(C) 400`.

Let $x_1, x_2, \ldots, x_{1000}$ be the students scores in the test. The average score is then given by

$$\frac{x_1 + x_2 + \ldots + x_{1000}}{1000} = 6.$$

Thus the sum of all the scores is $6 \cdot 1000 = 6000$. The maximum number of students that can could have scored 15 is then $\frac{6000}{15} = 400$, with the remaining 600 students scoring 0.

5 Question 5

5.1 Question

Alice and Bob both have a secret number. They don't tell anyone, not even each other. Bob's number is between 1 and 100 (included) and Alice's between 1 and 20 (included) and they know that their numbers are different. Yesterday I heard a conversation between them:

- Bob: "My number has (bzzzz) divisors."
- Alice: "Now I know your number!"
- Bob: "Interesting, now I know your number too!"

Unfortunately, I couldn't hear Bob's first statement in its entirety due to interference! What is Alice's number?

5.2 Answers

(A) 12

(B) 15

(C) 20

(D) 16

5.3 Proposed solution

The correct answer is `(D) 16`.

Let n be the number of divisors of Bob's number. Since Alice is able to immediately identify Bob's number, we have only two possibilities. Either there is only one number between 1 and 100 with n divisors, or there are two numbers and one of them is Alice's. In the first case Bob would't be able to get any information from the fact that Alice found his number, hence we are in the second case.
Looking at the possible answers we can see that 12 and 20 have both 6 divisors, as the number 28, so they cannot be correct. The number 15 has 4 divisors, as 6 and 10, so it also cannot be the answer. By process of elimination the answer is 16, which has 5 divisors. Indeed, between 1 and 100, the only numbers with 5 divisors are 16 and 81.
It is also possible, without looking at the answers, to extend the argument above to all numbers from 1 to 20 and see that the only possible answer is 16.

6 Question 6

6.1 Question

Four hackers are interviewed. We know that all of them are either black-hat or white-hat. Black-hat hackers always lie, while white-hat hackers always tell the truth:

- Hacker **One** said that exactly one of the four hackers is a black-hat
- Hacker **Two** said exactly two of the four hackers are black-hat.
- Hacker **Three** said that exactly three of the four hackers are black-hat.
- Hacker **Four** said that exactly four of the four hackers are black-hat.

How many of the hackers are black-hat?

6.2 Answers

(A) 0

(B) 1

(C) 2

(D) 3

6.3 Proposed solution

The correct answer is `(D) 3`.

Since all the sentences are contradictory, either all of them are false or only one is true. If all of them were false then hacker **Four** would be correct, which is a contradiction. Thus there is only one true sentence, which must be the third one, and hackers **One**, **Two** and **Four** are black-hat. Hence the answer is 3.

7 Question 7

7.1 Question

A special knight is moving on a board. The board is 15×15 and the knight can move 1 square in one direction and 3 squares in a perpendicular one (which is a diagonal of a 1×4 rectangle instead of a 1×3 like in chess). What is the maximum number of squares that a knight can reach within the board, among all the possible starting positions?

7.2 Answers

(A) 112

(B) 113

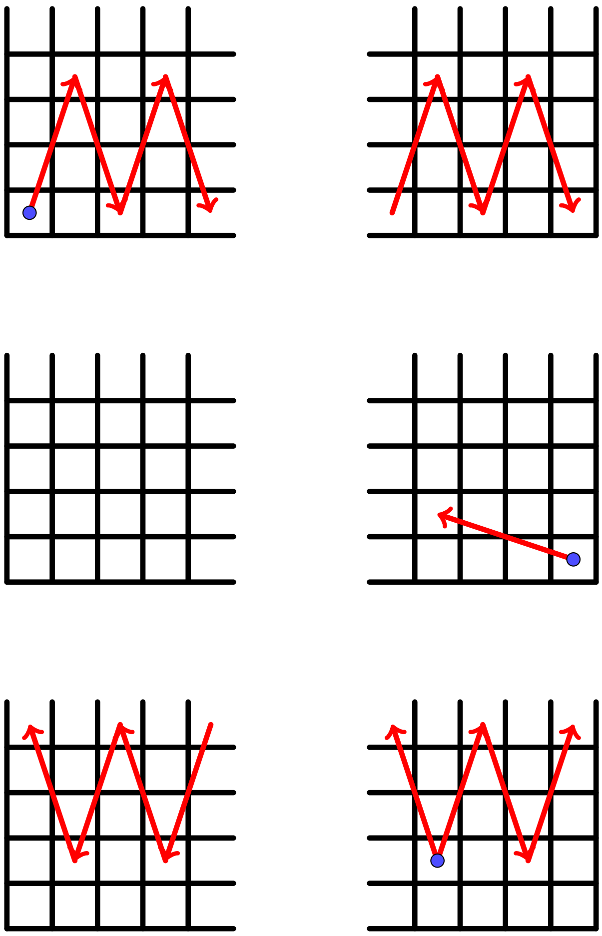
(C) 163

(D) 225

7.3 Proposed solution

The correct answer is `(B) 113`.

Considering the standard coloration of a board in white and black, we can observe that the horse always stays on the same color. Thus the horse can cover at most $\lceil \frac{15 \cdot 15}{2} \rceil = 113$ squares. We can achieve this by starting on the bottom left corner and moving up and down untill the bottom right corner. Then moving up one row and doing the same.



8 Question 8

8.1 Question

10 people are sitting in a circle. They are sitting such that each person is facing the back of the person in front of them. Each person picks either rock, paper, or scissors, with 2 people picking rock, 4 people picking paper, and 4 picking scissors.
A move consists of an operation of one of the following three forms:

- If a person picking rock sits behind a person picking scissors, they swap places.
- If a person picking paper sits behind a person picking rock, they swap places.
- If a person picking scissors sits behind a person picking paper, they swap places.

What is the maximum number of moves that can be performed, over all possible initial configurations?

8.2 Answers

(A) 4

(B) 10

(C) 16

(D) 32

8.3 Proposed solution

The correct answer is `(C) 16`.

Let's call S the number of triples of the form (rock, scissors, paper), in clockwise order. Let's consider a swap between two people A and B who picked x and y, with $x, y \in \{\text{rock}, \text{paper}, \text{scissors}\}$. After the swap, the number S of total triples described above will decrease at least by the number of people C who picked z, with $z \neq x, y$, since we are removing all the triples (A, B, C). Thus, at every swap, the number S of such triples will decrease at least of $\min(4, 4, 2) = 2$. Since the maximum number of such triples at the beginning is $4 \cdot 4 \cdot 2$, the maximum number of swaps that can be made is $4 \cdot 4 = 16$.
We can see that this is achievable by positioning all people picking rock behind the ones picking scissors behind the ones picking paper, in clockwise order, and swapping all scissors with all papers one at a time.

9 Question 9

9.1 Question

Lorenzo is writing numbers on a board. He starts writing the number 152. For 7 times he takes a random number x between $\{2^0, 2^1, 2^2, \dots, 2^7\}$ and substitutes the number n on the board with $n \oplus x$, where $\oplus$ is the bitwise xor operator. What is the probability that in the end the number written on the board is 255?
The bitwise XOR, denoted with $\oplus$, is the bit-by-bit operation defined by the following truth table: $0 \oplus 0 = 0, 1 \oplus 0 = 1, 0 \oplus 1 = 1, 1 \oplus 1 = 0$.

9.2 Answers

(A) 0

(B) $\frac{175}{2^{17}}$

(C) $\frac{405}{2^{17}}$

(D) $\frac{735}{2^{17}}$

9.3 Proposed solution

The correct answer is `(D)` $\frac{735}{2^{17}}$.

The number 152 in binary format is 10011000. Since xoring with a power of two is equivalent to flip the corresponding bit, to obtain $255 = 0b11111111$ in the end, Lorenzo has either to flip one time the 0 bits and two times one of the 1 bits, or flip three times one of the 0 bits and one time the others. Let's name $x_0, x_1, \dots, x_7$ the 8 bits of 152, with $x_0, x_3, x_4 = 1$ and the others equal to 0.
We can represent an extraction of of the 7 numbers as a string $x_{i_0}, x_{i_1}, \dots, x_{i_7}$ with $i_0, \dots, i_7 \in \{0, \dots, 7\}$. All the possible strings are 8^7, now let's count the ones we are interested on. In the first case we have that 5 of the values $i_0, \dots, i_7$ must be $\{1, 2, 5, 6, 7\}$ in some order, while the other two must be equal and in the set $\{0, 3, 4\}$. This is equivalent to counting the permutations of a string of 7 letters with two equal letters, multiplied by three for the three possible values in $\{0, 3, 4\}$. This is equal to $3 \cdot \frac{7!}{2!}$. The other case is similar, but with 5 possibilities for the 0 bit flipped three times and 3 equal characters in the word. In this case the possibilities are $5 \cdot \frac{7!}{3!}$. Thus the total number of favorable cases is

$$3 \cdot \frac{7!}{2!} + 5 \cdot \frac{7!}{3!} = 7! \cdot \left(\frac{3}{2} + \frac{5}{6}\right) = 7! \frac{14}{6} = 7^2 \cdot 5 \cdot 3 \cdot 16.$$

Hence the probability of obtaining 255 in the end is

$$\frac{7^2 \cdot 5 \cdot 3 \cdot 16}{2^{21}} = \frac{735}{2^{17}}.$$

10 Question 10

10.1 Question

There are 60 knights and knaves sitting on a round table, evenly distributed. Knaves always lie, while knights always tell the truth. Each person says "The two people next to me and the one in front of me are all knaves". How many knights are there at most?

10.2 Answers

(A) 25

(B) 29

(C) 30

(D) 31

10.3 Proposed solution

The correct answer is `(B) 29`.

Since the knights tell the truth, they cannot have another knight next to them. Thus they can at most be $\frac{60}{2} = 30$, but this is possible only if knights and knaves are alternated. Since $60 \mod 4 = 0$, in this case we would have that every knight has another knight in front of them, which is not possible.
We now show that it is instead possible to have 29 knights. Indeed, labelling the positions on the circle from 0 to 59, we can put knights and knaves alternated from 0 to 29, starting with a knight at position 0. We then put a knave at position 30 and start alternating again from 31 to 58 ending with another knave at position 59. This way every knight has two knaves next to him and one in front, while every knave always has a knight either in front or next to him.

11 Question 11

11.1 Question

Consider the following function:

```
function f(n){
    x = 1
    res = 0

    while (x < 100**12) {
        x = x*n
        res = res + 1
    }

    return res
}
```

Where `a**b` is the power function a^b. For how many integer values of `n` the function `f(n)` returns 3?

11.2 Answers

(A) 999999000000

(B) 999900000000

(C) 99999999

(D) 99000000

11.3 Proposed solution

The correct answer is `(B) 999900000000`.

The function f computes $f(n) = \lceil log_n(100^{12}) \rceil$. Thus, for $n < 100^4$ we have $f(n) \geq 4$, for $n \geq 100^6$ we have $f(n) \leq 2$ and for $100^4 \leq n < 100^6$ we have $f(n) = 3$. Hence the answer is $100^6 - 100^4 = 999900000000$.

12 Question 12

12.1 Question

Consider the following functions:

```
1  function encrypt(m, len){
2      for (i = 0; i < len - 1; i++) {
3          m[i] = chr((ord(m[i]) + ord(m[i+1])) % 26)
4      }
5
6      return m
7  }
8
9  function decrypt(m, len){
10     for (i = len-2; i > 1; i--) {
11         d = ord(m[i]) - ord(m[i+1])
12         c = chr((d + 26) % 26)
13         m[i] = c
14     }
15
16     return m
17 }
```

Where `ord` is the function that maps A to 0, B to 1 and so on, while `chr` is its inverse function. Assuming that there is exactly one error and that the encryption function is correct, which line of code is incorrect?

12.2 Answers

(A) 10

(B) 11

(C) 12

(D) 13

12.3 Proposed solution

The correct answer is `(A) 10`.

The encryption function adds to every character the next one modulo 26, starting from the first one end ending to the second to last, while the last one remains unchanged. Thus, the decryption function should subtract to every character the next one starting from the second to last and ending with the first one. But, at line 10 of the code, we can see that it ends the for loop at the third character, hence that line is incorrect.

13 Question 13

13.1 Question

Consider the following function:

```
function f(n){
    res = 0
    for (i = 0; i < n; i++) {
        res = (res << 1) + 1
    }

    return res

}
```

What is `f(n)` computing?

13.2 Answers

(A) `2*n`

(B) `2^n-1`

(C) `2*n-1`

(D) `2^n`

13.3 Proposed solution

The correct answer is `(B) 2^n-1`

At every iteration the function multiplies `res` by 2 and adds 1. Thus, the binary representation of the resulting number will have n consecutive 1s. Adding 1 to the result will lead to a number whose binary representation has a 1 and then n 0s, that corresponds to 2^n. Hence, the result of $f(n)$ is $2^n - 1$.

14 Question 14

14.1 Question

Consider the following functions:

```
function s(v, n){
    int t[n]
    for(i=0; i<n; i++){
        c = 0
        for(j=-1; j<2; j++)
            if(i+j >= 0 && i+j < n)
                c += v[i+j]
        if(c > 1)          t[i] = 0
        else if(c == 1)  t[i] = 1
        else               t[i] = v[i]
    }
    return t
}

function f(n){
    v = [1, 1, 0, 0, 0, 0, 0, 1, 1, 0]
    for(i=0; i<n; i++)
        v = s(v, 10)
    return v
}
```

What is the value of `f(1234567890)`?

14.2 Answers

(A) `[0, 0, 0, 0, 1, 0, 0, 0, 1, 0]`

(B) `[0, 0, 0, 0, 0, 1, 1, 1, 0, 0]`

(C) `[0, 0, 0, 1, 1, 1, 0, 1, 1, 1]`

(D) `[1, 0, 0, 0, 1, 0, 0, 0, 0, 0]`

14.3 Proposed solution

The correct answer is `(B) [0, 0, 0, 0, 0, 1, 1, 1, 0, 0]`

The function computes a simplified version of Conway's Game of Life is one dimension. By explicitly computing the function for small values of `n`, we can observe that it has an antiperiod of size 5 and a period of size 8:

```
f( 0) = [1, 1, 0, 0, 0, 0, 0, 1, 1, 0]
f( 1) = [0, 0, 1, 0, 0, 0, 1, 0, 0, 1]
f( 2) = [0, 1, 1, 1, 0, 1, 1, 1, 1, 1]
f( 3) = [1, 0, 0, 0, 0, 0, 0, 0, 0, 0]
f( 4) = [1, 1, 0, 0, 0, 0, 0, 0, 0, 0]
f( 5) = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
f( 6) = [0, 1, 1, 1, 0, 0, 0, 0, 0, 0]
f( 7) = [1, 0, 0, 0, 1, 0, 0, 0, 0, 0]
f( 8) = [1, 1, 0, 1, 1, 1, 0, 0, 0, 0]
f( 9) = [0, 0, 0, 0, 0, 0, 1, 0, 0, 0]
f(10) = [0, 0, 0, 0, 0, 1, 1, 1, 0, 0]
f(11) = [0, 0, 0, 0, 1, 0, 0, 0, 1, 0]
f(12) = [0, 0, 0, 1, 1, 1, 0, 1, 1, 1]
f(13) = [0, 0, 1, 0, 0, 0, 0, 0, 0, 0]
```

Hence, $f(x) = f(5 + ((x - 5)\%8)$ for $x \geq 5$. Since $1234567890 \mod 8 = 2$,

$$f(1234567890) = f(5+5) = f(10) = \texttt{[0, 0, 0, 0, 0, 1, 1, 1, 0, 0]}.$$

15 Question 15

15.1 Question

Consider the following function:

```
function f(l) {
        l = sort(l)
        count = 0
        for(int i = 0; i < len(l) - 1; i++){
                k = 0
                while(k < l[i+1] - l[0])
                        k += 1
                count += k
        }
        count += len(l) * l[0]
        return count
}
```

What does `f(l)` compute?

15.2 Answers

(A) Sum of the elements of `l`

(B) Product of the elements of `l`

(C) Maximum of the elements of `l`

(D) Average of the elements of `l`

15.3 Proposed solution

The correct answer is `(A) Sum of the elements of l`

Every cycle in the for loop adds `l[i+1] - l[0]` to `count`. Thus, exiting the for loop, `count` is equal to

$$\texttt{l[1]} + \ldots + \texttt{l[len(l) - 1]} - (\texttt{len(l)} - 1) \cdot \texttt{l[0]}.$$

Adding $\texttt{len(l)} \cdot \texttt{l[0]}$ we obtain

$$\texttt{count} = \texttt{l[0]} + \texttt{l[1]} + \ldots + \texttt{l[len(l) - 1]} = \texttt{sum(l)}.$$