CyberChallenge.IT 2020 - Test
Commented solutions

Contents

1 Question 1

1.1 Question

Which is the unit digit of the least common multiple between 4834 and 5135?

1.2 Answers

(A) 0

(B) 1

(C) 3

(D) 7

1.3 Proposed solution

The correct answer is `(A) 0`.

Let us recall that the least common multiple (or lcm) between two or more integers is the lowest integer that is multiple of all of the starting numbers[1].

In particular, 4834 is even and 5135 is multiple of 5, so the lcm of 4834 and 5135 is multiple of 10 (since it is multiple of both 2 and 5), and thus its unit digit is 0.

[1] See `https://en.wikipedia.org/wiki/Least_common_multiple`

2 Question 2

2.1 Question

What is the correct negation of this proposition? "For all green animals, it is true that if the green animal is also a frog, the animal likes water."

2.2 Answers

(A) There is a green animal that is a frog but does not like water

(B) All green animals that do not like water are not frogs

(C) Water is not green

(D) There is a frog that does not like water

2.3 Proposed solution

The correct answer is `(A) There is a green animal that is a frog but does not like water`.

Let us denote by G the set of green animals, with a a generic element of G, with p the proposition "a is a frog" and by q the proposition "a likes water". Then, the statement in the question is represented by

$$\forall a \in G : p \Rightarrow q$$

In order to negate a universal quantifier[2] (the "For all" expression), a counterexample must be provided. This means that the negation of the sentence should be

$$\exists a \in G : p \nRightarrow q$$

which, in natural language, is "There is a green animal that is a frog but does not like water".

[2] See `https://en.wikipedia.org/wiki/Universal_quantification#Negation`

3 Question 3

3.1 Question

Henry starts with a list of the first 1000 positive integers. At each step he performes the following operations:

- Erase all the non-positive integers, if any.
- Erase all the integers with repeated digits (e.g. 111, 121, 122), if any.
- Decrements every remaining number by 1.

After how many steps all the numbers will be erased?

3.2 Answers

(A) 100

(B) 11

(C) 10

(D) 101

3.3 Proposed solution

The correct answer is `(B) 11`.

Let us notice that the numbers that will remain longer inside the list are of the form $11k - 1$ for some integer k with $1 \leq k < 10$. In fact, in this case, the unit digit of the (original) number $11k - 1$ will cover all the ten possible values until the number, decreased at each iteration, will become $11(k - 1)$. For example, we have:

$$21 \to 20 \to 19 \to \cdots \to 12 \to 11$$

At the end of this process, the number will be erased.

It is clear that all the other numbers will be erased in less iterations, and this family of numbers will last 10 iterations, so that they will be erased at the 11^{th} one, when the list will become empty.

4 Question 4

4.1 Question

Consider the following function

```
function f(a,b,c) {
        if((not a) or b) return 1
        if(((not a) or (not b)) and c) return 2
        if(((not a) or b or (not b)) and a) return 3
        return 4
}
```

Which assignment of `(a,b,c)` makes `f(a,b,c)` equal to 3?

4.2 Answers

(A) `(false, true, false)`

(B) `(false, false, false)`

(C) `(false, false, true)`

(D) `(true, false, false)`

4.3 Proposed solution

The correct answer is `(D) (true, false, false)`.

Our goal is to not verify the first two `if` statements and verify the third one.

If `(not a) or b` should be false, then both `not a` and `b` are false. Therefore, `(not a) or (not b)` is true, so `c` must be false to avoid entering inside the second branch.

Finally, we can verify that `(a,b,c) = (true, false, false)` verifies the third condition, `((not a) or b or (not b)) and a`.

5 Question 5

5.1 Question

Consider the following function

```
function x(a,b) {
        if(b == 1) return a
        if(b % 2 == 1) return a*x(a*a, int(b/2))
        return x(a*a, int(b/2))
}
```

What does `x(3,3)` return?

5.2 Answers

(A) 9

(B) 6

(C) 27

(D) 3

5.3 Proposed solution

The correct answer is `(C) 27`.

The function `x` implements the square-and-multiply algorithm[3], which is an efficient method to compute powers of large numbers. This means that `x(a, b)` computes a^b, so `x(3, 3)` is $3^3 = 27$.

[3]See `https://en.wikipedia.org/wiki/Exponentiation_by_squaring`

6 Question 6

6.1 Question

Consider the following statements

- If Danny owns a computer also Mark owns a computer
- If Mark owns a computer, then also Steve owns a computer

Which of the following sentences are true if Danny owns a computer?

(a) Mark owns a computer

(b) Steve owns a computer

(c) Steve does not own a computer

6.2 Answers

(A) a and b

(B) Only b

(C) Only a

(D) Only c

6.3 Proposed solution

The correct answer is `(A) a and b`.

By the first statement, if Danny owns a computer, also Mark owns a computer. Moreover, by the second one, if Mark owns a computer, then also Steve owns a computer. Therefore, all of them own a computer, and in particular Mark and Steve do, so sentences `a` and `b` are true.

7 Question 7

7.1 Question

Consider the following functions

```
function x(n) {
        if(n <= 0) return 0
        return 1+y(n-1)
}

function y(n) {
        if(n <= 0) return 0
        return 1+x(n-2)
}
```

What does `x(100)` return?

7.2 Answers

(A) 33

(B) 67

(C) 68

(D) 69

7.3 Proposed solution

The correct answer is `(B) 67`.

Considering one call to `x` and one to `y`, the argument `n` is decreased by 3, while the (partial) return value is increased by 2. This means that with 33 calls to `x` (and 33 to `y`), the program is going to call `x(1)`, while the sum is 66. `x(1)` will add one more unit and calls `y(0)`, which stops the recursion and adds 0, for a total of 67.

8 Question 8

8.1 Question

Consider the following incomplete code fragment

```
for(i = 0; i < 5; i++) {
        for(j = 0; j < 5; j++) {
                if($1) print("x")
                else print("o")
        }
        print_new_line()
}
```

The expected output is:

```
ooxoo
oxxxo
xxxxx
oxxxo
ooxoo
```

What condition should replace `$1` to make the code correct?

8.2 Answers

(A) `(i + j) % 2 == 1`

(B) `-2 <= (i + j - 4) <= 2`

(C) `2 <= (i + j) <= 6`

(D) `0 <= (abs(2 - i) + abs(2 - j)) <= 2`

8.3 Proposed solution

The correct answer is `(D) 0 <= (abs(2 - i) + abs(2 - j)) <= 2`.

Let us analyze the different proposed answers:

- `(i + j) % 2 == 1`: with this condition, if `i = 0` and `j=1` the code will output an "x", while in the first row, second column of the output there is an "o";
- `-2 <= (i + j - 4) <= 2` with this condition, if `i = 0` and `j=3` the code will output an "x", while in the first row, fourth column of the output there is an "o";
- `2 <= (i + j) <= 6` this condition is equivalent to the one above;
- `0 <= (abs(2 - i) + abs(2 - j)) <= 2` with this condition, the output is exactly the expected one.

9 Question 9

9.1 Question

Let x, y and z be real numbers Consider the following equations:

$$x + y + z = 20$$
$$x + 2y + 3z = 16$$

What is the value of $x + 3y + 5z$?

9.2 Answers

(A) 10

(B) 8

(C) 12

(D) 4

9.3 Proposed solution

The correct answer is `(C) 12`.

Let us find, if exists, a linear combination[4] of the two equations that gives the third one, i.e. let us try to find k, h such that

$$x + 3y + 5z = k(x + y + z) + h(x + 2y + 3z).$$

The coefficients of the same variables can be analyzed separately, thus we can look at the x and the y coefficients to get the following system:

$$\begin{cases} k \cdot 1 + h \cdot 1 = 1 \\ k \cdot 1 + h \cdot 2 = 3 \end{cases}$$

whose solution is $(k, h) = (-1, 2)$.

We can verify that these valuse are valid also for the z coefficients, so we have

$$x + 3y + 5z = -(x + y + z) + 2(x + 2y + 3z) = -20 + 2 \cdot 16 = 12$$

[4]See `https://en.wikipedia.org/wiki/Linear_combination`

10 Question 10

10.1 Question

100 boxes are placed in a row. We know that exactly one of them contains a diamond.

Each box is labelled with “The diamond is either in the right box or in the left box”.

This is true for only one box.

How many boxes do we need to open to be sure to know where the diamond is?

10.2 Answers

(A) 2

(B) 1

(C) 98

(D) 99

10.3 Proposed solution

The correct answer is `(B) 1`.

If the label is true for only one box, then the box with the diamond has only one neighbour, otherwise two labels would tell the truth. This means that the diamond is inside one of the two boxes at the ends of the row. By opening one of them, we know exactly where the diamond is: inside the one we opened, if we see it, or inside the opposite one otherwise.

11 Question 11

11.1 Question

The numbers a,b,c,d are 1,2,2,3 in some order. What is the greatest possible value of $a^{b^{c^d}}$ (a to the b-th power to the c-th power to the d-th power)?

11.2 Answers

(A) 729

(B) 81

(C) 512

(D) 256

11.3 Proposed solution

The correct answer is `(C) 512`.

Let us notice that whatever is the power to which we elevate 1, it will allways remain a 1. So, in order to maximize $a^{b^{c^d}}$, we assign $d = 1$ (and we can forget about it since $x^1 = x$).

Now, we have to check for three values:

- $a = 2, b = 2, c = 3$, so that $a^{b^c} = 2^{2^3} = 256$
- $a = 2, b = 3, c = 2$, so that $a^{b^c} = 2^{3^2} = 512$
- $a = 3, b = 2, c = 2$, so that $a^{b^c} = 3^{2^2} = 81$

Now it is clear that the maximum value for the expression is 512.

12 Question 12

12.1 Question

You have 10 balls, 2 for each of 5 colours. The 2 balls of a certain colour weight exactly 1 gram more than the others, that have exactly the same weight, but you do not know the colour.

You are given a scale with two pans and a digital display that gives the signed differences between the two pans.

You can put whatever number of balls on each pan.

How many weighs do you need to determine the colour of the two different balls?

12.2 Answers

(A) 2

(B) 1

(C) 4

(D) 3

12.3 Proposed solution

The correct answer is `(B) 1`.

Let us denote with A, B, C, D, E the five colours. We can put one ball of colour A and two of colour B on one plate. Then, we put one ball of colour C and two balls of colour D on the other plate.

There are five possible outcomes:

- **the scale display shows -2**: then the heavier balls are of colour B
- **the scale display shows -1**: then the heavier balls are of colour A
- **the scale display shows 0**: then the heavier balls are of colour E
- **the scale display shows +1**: then the heavier balls are of colour C
- **the scale display shows +2**: then the heavier balls are of colour D

13 Question 13

13.1 Question

Consider the following statements

- Those who like programming like reversing
- Those who like guessing like forensics
- Those who do not like forensics do not like programming

Which of the following sentences are true?

(a) Those who like guessing like reversing

(b) Those who like programming like forensics

(c) Those who like reversing do not like guessing

(d) Those who like guessing do not like programming

(e) Those who like programming like guessing

13.2 Answers

(A) Only b

(B) b and c

(C) They are all false

(D) a and d

13.3 Proposed solution

The correct answer is `(A) Only b`.

Let us denote by p the proposition "this person does not like forensics" and by q the proposition "this person does not like programming". Then, the third statement in the question is represented by $p \Rightarrow q$ and, by the transposition rule[5], if $p \Rightarrow q$ is true, then also $\overline{q} \Rightarrow \overline{p}$ is true, which corresponds to the statement "Those who like programming like forensics".

This makes sentence `(b)` true, while all the other ones are not necessarily true (there might be examples for which the sentences are true and examples for which the same sentences are false, so they do not represent general rules).

[5] See `https://en.wikipedia.org/wiki/Transposition_(logic)`

14 Question 14

14.1 Question

On an island there are 2020 people, each of them are truth-tellers or liars. At a party, more than 2000 of them came and sat around a big circular table.

During the dinner, each of them said that his two neighbours are of different types (one truth-teller and one liar).

What is the maximum number of truth-tellers in the island?

14.2 Answers

(A) 1347

(B) 1353

(C) 1346

(D) 0

14.3 Proposed solution

The correct answer is `(B) 1353`.

Let us suppose that there exists at least one truth-teller at the table. Then, its neighbours should be another truth-teller and a liar; their other neighbours are a liar and a truth-teller respectively, and so on.

The table will look like the following, with T representing a truth-teller and L a liar:

$$\ldots - T - T - L - T - T - L - T - T - L - \ldots$$

So, 2 people over 3 among the ones at the table are truth-tellers. In order to get the maximum number of truth-tellers, we can consider 2001 people at the table (since more than 2000 came to the party), with a total of $2001 \cdot \frac{2}{3} = 1334$ truth-tellers, plus the other 19 that we can assume truth-tellers, given that no information about them is given. The maximum number of truth-tellers is thus $1334 + 19 = 1353$.

15 Question 15

15.1 Question

Consider the following function

```
function f(int x, int y) {
        if(y == 0) return x
        return f(x^y, 2*(x&y))
}
```

Where `^` and `&` are bitwise XOR and bitwise AND operations.
What operation is equivalent to the function `f`?

15.2 Answers

(A) `x * y`

(B) `x - y`

(C) None of the others

(D) `x + y`

15.3 Proposed solution

The correct answer is `(D) x + y`.

Let us show that `f` is computing the value of `x + y`.

In binary, the addition of two bits behaves in the following way:

x	y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Let us notice that the column of the sum is the same as the one of the bitwise XOR, while the carry behaves like the bitwise AND.

Finally, the multiplication by 2 is needed to sum each carry with the bits in the correct position.

To sum up, the function is recursively doing the sum without carries (the XOR), calculating the bit array of carries (the AND and the multiplication by 2) and repeating the whole process until there is no carry, thus adding the two values `x` and `y`.